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ANOTHER UNIFORM ASSOCIATION MODEL APPLIED TO CAR ACCIDENT DATA OF 2x3 TABLE

by

Sadao Tomizawa
Department of Information Sciences,
Faculty University of Science & Technology,
Science of Tokyo, Noda City
Chiba 278 Japan

Abstract

This note proposes another version of Goodman's (1979) uniform association model for a 2x3 contingency table. The proposed model is an extension of the independence model. The model is applied to car accident data cross-classified according to the accient type and the accident severity.

Keywords: independence model; odds-ratio.

1. Introduction

Consider a 2x3 contigency table with cell probabilities (p_{ij}). The uniform association (UA) model is defined by

$$p_{ij} = \alpha_i \beta_j \theta^{(i-1)(j-1)} \qquad \text{for} \quad i=1,2; j=1,2,3$$

where $\alpha_1=1$ without loss of generality (see Goodman, 1979). A special case of this model obtained by putting $\theta=1$ is the usual independence (IND) model. [Note that Goodman (1979) referred to the IND model as the null association model.]

Define three odds-ratios as

$$\theta_1 = \frac{p_{11}p_{22}}{p_{12}p_{21}}, \quad \theta_2 = \frac{p_{12}p_{23}}{p_{22}p_{13}} \text{ and } \theta_3 = \frac{p_{11}p_{23}}{p_{21}p_{13}} \; (=\theta_1\theta_2).$$

Then the IND and UA models may be expressed as $\theta_1=\theta_2=1(=\theta_3)$ and $\theta_1= \theta_2$, respectively (see Goodman, 1979).

Let f_{ij} denote the observed frequency in the cell (i,j) of the 2x3 table (i=1,2;j=1,2,3), and let $\{\theta_K^*\}$, k=1,2,3, denote $\{\theta_K\}$ with $\{p_{ij}\}$ replace by $\{f_{ij}\}$. To demonstrate $\{\theta_k^*\}$, consider the data in Table 1. Table 1 taken from Read & Cressie (1988, p.20) shows the data on the 4831 car accidents which are cross-classied according to the accident type and the accident severity. For these data, $\theta_1^* =5.89$, $\theta_2^*=1.08$ and $\theta_3^* =6.36$ $(=\theta_1^* \theta_2^*)$. Therefore, for these data, (i) the IND model is unlikely to hold because θ_1^* and θ_3^* are not close to one, and also (ii) the UA model is unlikely to hold because θ_1^* is not close to θ_2^* (also see Table 2 and Section 3). However, we see that θ_1^* is very to θ_3^*. Therefore we are interested in applying a model with the structure of $\theta_1= \theta_3$ to these data.

The purpose of this note is (i) to propose another UA model, which has a structure of $\theta_1 = \theta_3$ and is an extension of the IND model, and (ii) to analyze the car accident data using the new model.

2. Another uniform association model

Consider a model defined by

$$p_{ij} = \begin{cases} \alpha_i\beta_j\theta & \text{for } i=2;j=2,3, \\ \alpha_i\beta_j & \text{otherwise,} \end{cases} \qquad (2.1a)$$

where $\alpha_1=1$ without loss of generality. A special case of this model obtained by putting $\theta=1$ in the IND model. Model (2.1a) may be also expressed as

$$\theta_1 = \theta_3 \quad (=\theta) \quad [\text{or as } \theta_2 = 1]. \qquad (2.1b)$$

This shows that (2.1) is another version of the UA model. Therefore we shall refer to (2.1) as another UA (AUA) model. Let X and Y denote the row and column variables, respectively. From (2.1b), the AUA model states that the odds that $Y=j$ ($j=2,3$) instead of $Y=1$ is θ times higher when $X=2$ rather than when $X=1$.

Assume that a multinomial distributin applies to the 2x3 table. The maximum likelihood estimates (MLEs) of expected frequencies $\{m_{ij}\}$ under the AUA model are $m_{11}=f_{11}$, $m_{21}=f_{21}$, $m_{ij} = (f_{i2}+f_{i3})(f_{1j}+f_{2j})/(f_{12}+f_{13}+f_{22}+f_{23})$ for $i=1,2; j=2,3$. The goodness of fit of the AUA model can be tested by a chi-squared statistic with one degree of freedom.

3. Analysis of car accident data

Both of the IND and UA model fit the data in Table 1 very poorly (see Table 2). However, the AUA model fits these data very well (see Table 2). For testing the hypothesis that the IND model holds under the assumption that the AUA model holds true (namely, the hyphotesis that $\theta=1$ under the assumption), the difference between the likelihood ratio chi-squared statistics for the IND and AUA models is 601.42 with 2-1=1 degree of freedom. Therefore this hypothesis is rejected at the 0.01 level. Thus we can see the very strong evidence of $\theta\neq1$ in the AUA model (namely the very strong effect of parameter θ in the AUA model).

Under the AUA model applied to these data, the MLE of θ is 6.03. Therefore the AUA model applied to these data provides that (i) the odds that the accident severity of a car is "Moderately severe" instead of "Not severe" is estimated to be 6.03 times higher when the accident type of the car is "Not rollover" rather than when it is "Rollover", and (ii) the odds that the accident severity of a car is "Severe" instead of "Not severe" is estimated to be identically 6.03 times higher when the accident type of the car is "Not rollover" rather than when it is "Rollover".

Table 1

The 4831 car accidents cross-classified according to accident type and accident severity; from Read & Cressi (1988, p.20). The parenthesized values are the maximum likelihood estimates of expected frequencies under the AUA model.

	Accident severity			
Accident type	Not severe	Moderately severe	Severe	Total
Rollover	2365 (2365.0)	944 (935.2)	412 (420.8)	3721
Not rollover	249 (249.0)	585 (593.8)	276 (267.2)	1110
Total	2614	1529	688	4831

Table 2

Likelihood ratio chi-squared values for models applied to Table 1

Applied models	Degrees of freedom	Likelihood ratio chi-squared
IND	2	602.11
UA	1	133.18
AUA	1	0.69

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